Commitment Optimization

Description

This document describes Aurora's mathematical formulation of the commitment optimization problem. The commitment optimization logic uses a mixed-integer program (MIP) to determine the hourly unit commitment and dispatch pattern that will minimize total system cost subject to various constraints. The model simultaneously solves one day plus some number of additional look-ahead days defined by the user. Commitment decisions are modeled using binary variables, and commitment constraints of minimum up and down times are part of the formulation. Resource capability, transmission capacity, and demand constraints are honored in addition to many user-defined constraints such as fuel, hydro, storage, ancillary services, and resource dependencies. Resource dispatch and start costs as well as transmission wheeling costs are used to formulate the objective function. An input tolerance (MIP gap) is specified by the user, and the final solution is guaranteed to be within this tolerance of the optimal least-cost solution.

Mathematical Framework

Notation

Let $R_1, R_2, ..., R_n$ be a given set of resource segments, and let $Z_1, Z_2, ..., Z_m$ be a given set of zones. Each zone defines a set of resource segments belonging to it. Note that each resource must belong to exactly one zone. Also note that in the mathematical notation below (with the exception of subscripts and indices), all lower-case letters represent decision variables and all upper-case letters represent data input by the user or derived beforehand by the model.

Let M be the set of all minimum resource segments, and let H be the set of all dispatchable (non-minimum) resource segments. Define the following:

$p \in \{1, 2, ..., P\}$ is an index denoting the hour (or period) being solved
h is an index denoting the resource
$R^{(h)}$ is the set of all resource segments belonging to resource h
k is an index denoting the day
A_k is the set of all hours p in day k
r_i^p represents the dispatch for resource segment R_i in hour p (decision variables)
t_{ij}^p = transmission from Z_i to Z_j in hour p (decision variables)
U_i^p = capability of resource segment R_i in hour p
C_i^p = dispatch cost of resource segment R_i in hour p
G_i^p = start-up cost for the resource associated with minimum segment $R_i \in M$ in hour p
$s_h^p = 1$ if resource h started up in hour p, 0 if not (decision variables)
$d_h^p = 1$ if resource h decommitted in hour p, 0 if not (decision variables)
E_h = minimum up-time in hours for resource h
F_h = minimum down-time in hours for resource h
T_{ij}^p = transmission capacity from Z_i to Z_j in hour p
L_{ij}^p = loss factor for transmission from Z_i to Z_j in hour p
W_{ij}^p = wheeling charge for transmission from Z_i to Z_j in hour p
D_j^p = demand for zone Z_j in hour p

MIP Formulation

The model creates the following objective function to represent the total cost of the system. Total cost is the sum of total resource costs (start-up and dispatch) and total wheeling costs, summed over all hours:

$$\sum_{p=1}^{P}\left(\sum_{R_i\in M}(G_i^p s_i^p + r_i^p U_i^p C_i^p) + \sum_{R_i\in H} r_i^p C_i^p + \sum_{i=1}^{m}\sum_{j=1}^{m} t_{ij}^p W_{ij}^p\right) \tag{1}$$

Bounds on the resource variables ensure that minimum and maximum energy requirements are obeyed. That is, for all $i \in \{1,2,...,n\}$ and for all $p \in \{1,2,...,P\}$:

If $R_i \in M$ and R_i is a must run unit in hour p, then

$$r_i^p = 1 \tag{2}$$

If $R_i \in M$ and R_i is a commitment unit in hour p, then

$$r_i^p \in \{0,1\} \tag{3}$$

If $R_i \in H$, then

$$0 \leq r_i^p \leq U_i^p \tag{4}$$

Transmission constraints require that transmissions between zones are within capacity. For all $i,j \in \{1,2,...,m\}$ (with $T_{ij}=0$ for $i=j$) and for all $p \in \{1,2,...,P\}$:

$$0 \leq t_{ij}^p \leq T_{ij}^p \tag{5}$$

Demand constraints require that total supply equals total demand in each zone for each hour. For all $j \in \{1,2,...,m\}$ and for all $p \in \{1,2,...,P\}$:

$$D_j^p = \sum_{R_i\in Z_j\cap M} r_i^p U_i^p + \sum_{R_i\in Z_j\cap H} r_i^p + \sum_{i=1}^{m}(L_{ij}^p t_{ij}^p - t_{ji}^p) \tag{6}$$

Time constraints ensure that resource minimum up- and down-time requirements are obeyed. Let q_h^p represent resource h's minimum segment in hour p. The first two constraints set the values for the resource's start and decommit variables, s_h^p and d_h^p. The last two constraints actually enforce the up- and down-time requirements. The constraints must hold for all commitment resources h and all hours p.

$$s_h^p \geq q_h^p - q_h^{p-1} \tag{7}$$

$$d_h^p \geq q_h^{p-1} - q_h^p \tag{8}$$

$$\left(\sum_{i=1}^{E_h-1} s_h^{p-i}\right) - q_h^p \leq 0 \tag{9}$$

$$\left(\sum_{i=1}^{F_h-1} d_h^{p-i}\right) + q_h^p \leq 1 \tag{10}$$

Minimum segment constraints ensure that the MIP does not dispatch a resource's non-minimum segment without also dispatching its minimum segment. Let q_h^p represent resource h's minimum segment in hour p. Then, the following constraint must hold for all resources h and all hours p:

$$\sum_{R_i \in R^{(h)} \cap H} r_i^p \leq q_h^p (\sum_{R_i \in R^{(h)} \cap H} U_i^p) \tag{11}$$

To summarize, the model formulates the following MIP and finds the vectors $\boldsymbol{r^p}$ and $\boldsymbol{t^p}$ of resource dispatch and transmission between zones, respectively, for each hour $p \in \{1, ..., P\}$:

$$\begin{array}{lll}
\text{minimize} & \sum_{p=1}^{P} \left(\sum_{R_i \in M} (G_i^p s_i^p + r_i^p U_i^p C_i^p) + \sum_{R_i \in H} r_i^p C_i^p + \sum_{i=1}^{m} \sum_{j=1}^{m} t_{ij}^p W_{ij}^p \right) & \\
\text{subject to} & r_i^p = 1 \qquad R_i \in M \text{ is a must run unit in hour } p & \\
& r_i^p \in \{0, 1\} \qquad R_i \in M \text{ is a commitment unit in hour } p & \\
& 0 \leq r_i^p \leq U_i^p \qquad R_i \in H & \\
& 0 \leq t_{ij}^p \leq T_{ij}^p & \text{for all } i, j, \text{ and } p \\
& D_j^p = \sum_{R_i \in Z_j \cap M} r_i^p U_i^p + \sum_{R_i \in Z_j \cap H} r_i^p + \sum_{i=1}^{m} (L_{ij}^p t_{ij}^p - t_{ji}^p) & \text{for all } j, p \\
& s_h^p \geq q_h^p - q_h^{p-1} & \text{for all } h, p \\
& d_h^p \geq q_h^{p-1} - q_h^p & \text{for all } h, p \\
& \left(\sum_{i=1}^{E_h - 1} s_h^{p-i} \right) - q_h^p \leq 0 & \text{for all } h, p \\
& \left(\sum_{i=1}^{F_h - 1} d_h^{p-i} \right) + q_h^p \leq 1 & \text{for all } h, p \\
& \sum_{R_i \in R^{(h)} \cap H} r_i^p \leq q_h^p (\sum_{R_i \in R^{(h)} \cap H} U_i^p) & \text{for all } h, p
\end{array}$$

Additional Constraints

In addition to the global constraints outlined above, the user may also impose additional hourly and/or daily constraints on the model. These include hourly fuel limits, generation MW constraints, multilink limits, and multiple fuel constraints as already defined in the document "Zonal LP Dispatch." In addition, the following constraints can also be imposed when using the commitment optimization logic:

Daily Fuel: This places a daily cumulative fuel limit on all resources that utilize the specified fuel directly or indirectly. Let X MMBtu be the limit for day k. Let S be the set of all resource segments that utilize the fuel, and call it fuel f. Let $r_{i_f}^p$ denote the output of resource segment R_i generated by utilization of fuel f in hour p. Let Q_i^p denote the heat rate of resource segment R_i in Btu/kWh in hour p.

$$\sum_{p \in A_k} \left(\sum_{R_i \in S \cap M} \frac{Q_i^p r_{i_f}^p U_i^p}{1000} + \sum_{R_i \in S \cap H} \frac{Q_i^p r_{i_f}^p}{1000} \right) \leq X \tag{12}$$

Daily Hydro: This places a daily requirement on a hydro resource, while allowing hours to remain flexible within the dispatch. Let X MWh be the allocated hydro energy for day k as determined by the standard monthly hydro shaping logic. Let w indicate the resource with its corresponding set $R^{(w)}$ of resource segments.

$$\sum_{p \in A_k} \left(\sum_{R_i \in R^{(w)} \cap M} r_i^p U_i^p + \sum_{R_i \in R^{(w)} \cap H} r_i^p \right) \leq X \tag{13}$$

Daily Storage: This places a daily requirement on a storage resource. Let X MW be the net change to the storage contents of the resource for day k as determined by the week-ahead storage scheduling algorithm. Let s indicate the resource with its corresponding set $R^{(s)}$ of resource segments. Let e^p denote the MW amount of energy stored by the resource in hour p, and let g_i^p denote the MW amount of energy generated by resource segment R_i in hour p. The generation must then satisfy the following constraint.

$$\sum_{p \in A_k} \left(\sum_{R_i \in R^{(s)} \cap H} g_i^p - e^p \right) = X \tag{14}$$

The model also adds constraints that ensure that the pond level stays between 0 and the maximum storage level, that the hourly generation charging patterns are within the required limits, and that the proper load increase occurs on the system when the unit is pumping.

Hourly Ancillary Services: There are two types of ancillary products. Up type products require that enough excess capacity be online or available within a short time horizon to ramp up to meet unexpected load increases. Down type products require that enough capacity be online and available to ramp down within a short time horizon to meet unexpected load decreases. Let X MW be the ancillary requirement for a given product in hour p. Let S be the set of all resources assigned to the ancillary services product. Let q_h^p denote the minimum segment for resource h in hour p. Furthermore, assume that a resource cannot contribute if offline, and that each resource h assigned to the product has a minimum capability $R_{min}^{(h)p}$ and maximum capability $R_{max}^{(h)p}$ in hour p. Let c_h^p denote the MW amount that resource h can contribute to the ancillary product in hour p.

- For an up type product, the contribution for each resource is limited by the following:

$$c_h^p \leq \left(R_{max}^{(h)p} - R_{min}^{(h)p}\right) q_h^p - \sum_{R_i \in R^{(h)} \cap H} r_h^p \tag{15}$$

- For a down type product, the contribution for each resource is limited by the following:

$$c_h^p \leq \sum_{R_i \in R^{(h)} \cap H} r_h^p \tag{16}$$

The overall ancillary constraint is then given below.

$$\sum_{R^{(h)} \in S} c_h^p = X \tag{17}$$

Hourly Operating Rule

- Committed: This requires that the number of committed resources in a given set must be less than, equal to, or greater than a number X in hour p. Let S be the set of all resource segments belonging to the resources in the given set.

$$\sum_{R_i \in S \cap M} r_i^p \quad \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} \quad X \tag{18}$$

- Committed MW: This requires that, in a given set S of resources, there must be less than, equal to, or greater than X MW of committed capability in hour p. Let V_h^p represent that total capability of resource h in hour p.

$$\sum_{R_i \in S \cap M} r_i^p V_h^p \quad \left\{ \begin{matrix} \leq \\ = \\ \geq \end{matrix} \right\} \quad X \tag{19}$$

Hourly Ramp Rates

- Ramp Rate: This ensures that a unit cannot increase its output by more than X MW in hour p. Let h indicate the given resource with its corresponding set $R^{(h)}$ of resource segments.

$$\sum_{R_i \in R^{(h)} \cap M} (r_i^p U_i^p - r_i^{p-1} U_i^{p-1}) + \sum_{R_i \in R^{(h)} \cap H} (r_i^p - r_i^{p-1}) \leq X \tag{20}$$

- Ramp Down Rate: This ensures that a unit cannot decrease its output by more than X MW in hour p. Again, let h indicate the given resource with its corresponding set $R^{(h)}$ of resource segments.

$$\sum_{R_i \in R^{(h)} \cap M} (r_i^{p-1} U_i^{p-1} - r_i^p U_i^p) + \sum_{R_i \in R^{(h)} \cap H} (r_i^{p-1} - r_i^p) \leq X \tag{21}$$

Hourly Resource Dependency: These constraints ensure that, in hour p, if the condition(s) defined for the independent unit(s) is (are) met, then the condition defined for the dependent unit must be satisfied as well. There are several independent and dependent conditions. Below, constraints are detailed for independent conditions of "Output > 0", "Not Committed", "Full Output", and "Not Full Output", and dependent conditions of "Output > 0". Other options within Aurora include the "Available" and "Unavailable" independent conditions and "Unavailable" dependent condition. Note that although the constraints below are written in terms of a single independent unit, in Aurora the user may define an entire set of independent units with varying conditions joined by AND or OR.

For the constraints below, let $R^{(ind)}$ denote the set of resource segments belonging to the independent unit, and let $R^{(dep)}$ denote the set of resource segments belonging to the dependent unit. Let q_{ind}^p and q_{dep}^p be the variables representing the minimum segments of the independent and dependent resources in hour p, respectively. Let U_{ind}^p be the capability of the independent resource, and let $U_{q(ind)}^p$ be the capability of the minimum segment of the independent resource. When a constraint is in use, it must hold for all hours p.

- Independent Output $> 0 \implies$ Dependent Output > 0: If the independent unit is committed, then the dependent unit must be committed as well.

$$q_{ind}^p \leq q_{dep}^p \tag{22}$$

- Independent Not Committed $\implies$ Dependent Output > 0: If the independent unit is not committed, then the dependent unit must be committed.

$$1 - q_{ind}^p \leq q_{dep}^p \tag{23}$$

- Independent Full Output $\implies$ Dependent Output > 0: If the independent unit is at full capability, then the dependent unit must be committed. If output divided by capability for a given unit is greater than 0.999, then within Aurora the unit is considered to be at full capability.

$$\left(\frac{r_{ind}^p U_{ind(q)}^p + \sum_{R_i \in R^{(ind)} \cap H} r_i^p}{U_{ind}^p} \right) - 0.999 \leq q_{dep}^p \tag{24}$$

- Independent Not Full Output $\implies$ Dependent Output > 0: If the independent unit is not at full capability, then the dependent unit must be committed. Recall that if output divided by capability for a given unit is greater than 0.999, then within Aurora the unit is considered to be at full capability.

$$0.999 - \left(\frac{r_{ind}^p U_{ind(q)}^p + \sum_{R_i \in R^{(ind)} \cap H} r_i^p}{U_{ind}^p} \right) \leq q_{dep}^p \tag{25}$$